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Department of Mathematics
B.Sc. II (Semester-IV)
Algebra II (BMT-402)
Subject Code: 16004

1: Answer in one sentence

1) Cyclic group

2) Index of a subgroup

3) Quotient group

4) Euler's $\emptyset$ function

5) Cosets of subgroup

6) Simple group

7) Order of an element of a group

8) Kernel of a homomorphism

9) Congruence relation

10) Centre of a group

11) Subgroup

12) Normalizer of an element $'a'$ of a group

13) Generator of cyclic group

14) Normal subgroup

15) Proper normal subgroup

16) Homomorphism of groups

17) Isomorphism

18) Endomorphism

19) Monomorphism

20) Group

21) Abelian group

22) Permutation group

23) Automorphism

24) Transposition

25) Equivalence class

2. Long answer questions

1) Show that if G is a finite group and H is a subgroup of G then

   $o(H)$ divides $o(G)$.

2) Prove that for any integer a and prime $p > 0$ then $a^p \equiv a(modp)$.

   Find the remainder of 3^{47} when divided by 23.

3) If $f: G \to G'$ is a homomorphism. Show that

   $i)\ f(e) = e'$.

   $ii)\ f(x^{-1}) = [f(x)]^{-1}$.

   $iii)\ f(x^n) = [f(x)]^n$, n is an integer.

   Where e, e' are identity elements of $G,\ G'$ respectively.

4) Show that a non empty subset H of a group G is a subgroup of G if and

   only if

   $i)\ a,\ b \in H \Rightarrow ab \in H$

   $ii)\ a \in H \Rightarrow a^{-1} \in H$

5) Prove that order of a cyclic group is equal to the order of its generator.

6) If mapping $f: G \to G'$ be an onto homomorphism with $K = kerf$ then show

   that $\frac{G}{K} \cong G'$.

7) Show that a non empty subset H of a group G is a subgroup of G if and only

   if $a, b \in H \Rightarrow ab^{-1} \in H$.

8) Prove that subgroup of a cyclic group is cyclic.

9) Show that a subgroup H of a group G is normal in G if and only if

   $g^{-1}hg \in H$ for all $h \in H,\ g \in G$.

10) Let H and K be two subgroups of group G, where H is normal in G

then show that $\frac{HK}{H} = \frac{K}{H \cap K}$.

11) If H and K are two normal subgroups of group G such that $H \subseteq K$

then show that $\frac{G}{K} \cong \frac{G/H}{K/H}$.

12) Show that every group G is isomorphic to a permutation group.

13) Show that a subgroup H of a group G is a normal subgroup of G if and only if product of two right cosets of H in G is again a right coset of H in G.

14) Prove that for any integer a and prime $p > 0$ then $a^p \equiv a(mod p)$.

Find the remainder of 4^{107} when divided by 13.

15) State and prove Lagrange's theorem.

3. Short answer questions

1) Show that if H_1 and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a subgroup of G.

2) Show that an infinite cyclic group has precisely two generators.

3) Show that a subgroup H of a group G is normal in G if and only if $g^{-1}Hg = H$ for all $g \in G$.

4) Show that the intersection of any two normal subgroups of a group is a normal subgroup.

5) Show that a homomorphism $f: G \to G'$ is one-one if and only if $kerf = \{e\}$.

6) For a finite group G, show that order of any element of G divides order of G.

7) Let H be a subgroup of G. Show that $Ha = H$ if and only if $a \in H$.

8) Show that centre of a group G is a subgroup G.

9) By using Fermat's theorem find the remainder of 8^{103} when divided by 13.

10) Show that every subgroup of an abelian group is normal.

11) By using Fermat's theorem find the remainder of 4^{107} when divided by 13.

12) Show that every quotient group of a cyclic group is a cyclic.

13) Let $< Z, +>$ and $< E, +>$ be the groups of integers and even integers. Define mapping $f: Z \to E$ such that $f(x) = 2x$ for all $x \in Z$. Show that f is isomorphic.

14) Show that normalizer of $a \in G$ is subgroup of G.

15) Let H be a subgroup of G. Show that $Ha = Hb$ if and only if $ab^{-1} \in H$.

16) By using Fermat's theorem find the remainder of 3^{47} when divided by 23.

17) Prove that for any integer a and prime $p > 0$ then $a^p \equiv a(modp)$.

18) By using Euler's theorem, find the remainder of 2^{48} when divided by 105.

19) Show that HK is a subgroup of G if and only if $HK = KH$.

20) Show that $Ha = \{x \in G |\, x \equiv a(modH)\}$ for any $a \in G$.

21) Show that Ha is a subgroup of G if and only if $a \in H$.

22) Show that centre of a group G is a subgroup of G.

23) Let a be an element of group G. Show that the set H of all integral powers of $'a'$ is a subgroup of G.

24) Show that every quotient group of an abelian group is abelian.

25) Show that every quotient group of a cyclic group is a cyclic.

26) G is finite group and N is a normal subgroup of $'G'$ then show that $o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$.

27) Let N be a normal subgroup of a group then show that $\frac{o(Na)}{o(a)}$ for any $a \in G$.

28) Show that any infinite cyclic group is isomorphic to the group of integers.

29) Suppose G is a group and N is a normal subgroup of G. Let $f: G \to \frac{G}{N}$ defined by $f(x) = Nx,$ for $x \in G$. Show that f is homomorphism of G onto $\frac{G}{N}$.

30)Show that the mapping $f: Z \to Z$ such that $f(x) = -x$ for all $x \in Z$ is an automorphism of the additive group of integers z.