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Department of Mathematics
B.Sc.-III(Semester-VI) MATHEMATICS
Complex Analysis (BMT 603)
Subject Code: 16012

Que 1. Answer the following.

1) Write Cauchy- Riemann equations in cartesian form.
2) Define singular point
3) How to calculate the singularity of function f(z) at infinity?
4) Define pole.
5) What is the isolated singularity?
6) Define the Jordan curve.
7) Define complex valued function.
8) Define meromorphic function
9) Define limit of a complex valued function
10) Write Cauchy- Riemann equations in polar form.
11) Define principal argument of complex number
12) Define harmonic function
13) What is the modulus and argument of $z = -i$.
14) Define removable singularity.
15) Define regular point.
16) Define simple curve.
17) Write the solution of exact differential equation $Mdx + Ndy = 0$.
18) Define entire function.
19) Write the equation of circle whose center is at c and radius a in complex plane.
20) Define analytic function.
21) Write statement of Greens theorem.
22) Define essential singularity.
23) Define cross cut.
24) Define smooth curve.

25) Define residue of an isolated singularity.

Que. 2 Solve the following questions.

1) Show that $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

2) If $f(z) = u + iv$ is an analytic function. If $f'(z) = 0$ then show that $f(z)$ is constant.

3) If $f(z) = u + iv$ is an analytic function then show that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z\ \partial \bar{z}}$.

4) If $u = e^x\, cosy$ is harmonic function then find the harmonic conjugate of u by using exact differential equation method and hence construct the analytic function.

5) Evaluate $\int \bar{z}\, dz$ along the line from $z = 0$ to $z = 2i$ and then from $z = 2i$ to $z = 4 + 2i$.

6) Find the value of integral $\int_0^{1+i} (x - y + ix^2)dz$ along the straight-line from $z = 0$ to $z = 1 + i$.

7) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $|z| > 3$.

8) Show that $\int_C \bar{z}\, dz$ along a semi-circular path from $z = -a$ to $z = a$ lies above the x axis is $-\pi i a^2$.

9) If $f(z) = \frac{z-4}{(z-3)\,(z-5)^2}$ then find the residues at corresponding poles.

10) Prove that analytic function with constant modulus is constant.

11) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

12) Show that the real and imaginary part of $f(z) = e^z$ are harmonic.

13) Evaluate $\int_0^{2\pi} \frac{1+2\, cos\theta}{5+4\, cos\theta}\, d\theta$.

14) If $f(z) = u + iv$ is an analytic function then show that $u(x, y) = c_1$ and $v(x, y) = c_2$ represent orthogonal family of curves.

15) Evaluate $\int_0^{1+i} z^2\, dz$.

16) If $f(z) = u + iv$ is an analytic function then find $f(z)$ in terms of z where $u - v = (x - y)(x^2 + 4xy + y^2)$.

17) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $1 < |z| < 3$.

18) If $f(z) = \frac{z^4}{z^2+a^2}$ then find the residues at corresponding poles.

19) Show that $\int_C \bar{z}\, dz$ along a semi-circular path from $z = -a$ to $z = a$ lies below the x axis is $\pi i a^2$.

20) Find the type of singularities of $f(z) = \frac{cot\pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$.

21) If $u = log(x^2 + y^2)$ then construct the analytic function.

22) Evaluate $\int_C \frac{z^4}{z-3i}\, dz$ by using Cauchy integral formula along the curve $|z-2| < 5$.

23) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

24) What kind of singularities exist for the function $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$.

25) Find the value of integral $\int_0^{1+i} (x - y + ix^2)dz$ along the straight-line from $z = 0$ to $z = 1 + i$.

26) Find the residue of $f(z) = \frac{z^2}{(z^2+1)^2}$ at $z = \infty$.

27) Expand $f(z) = \frac{1}{z\,(z^2-3z+2)}$ in a Laurent's series for the region $0 < |z| < 1$.

28) If $f(z) = \frac{z-4}{(z-3)\,(z-5)^2}$ then find the residues at corresponding poles.

29) Evaluate $\int_C \frac{e^{az}}{z+1}\, dz$ over the circle $C: |z| = 2$.

30) Find the type of singularities of $f(z) = tan\left(\frac{1}{z}\right)$ at $z = 0$.

Que. 3 Solve the following questions. (long answers)

1) State and prove necessary conditions for f(z) to be analytic.
2) If $f(z) = u(x, y + iv(x, y)$ is analytic function and $z = re^{i\theta}$ is a polar form of z then show that Cauchy- Riemann equation in polar form are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$
3) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles $z_1, z_2, \ldots, z_n$ within the closed contour and continuous on boundary which is a

rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^{n} Res(z = z_k)$ hence evaluate $\int_C \frac{dz}{z^3(z+3)}$ over the circle $C: |z| = 1$.

4) Explain exact differential equation method for construction of an analytic function. Hence construct analytic function for $u = e^x \, cosy$

5) If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C, if $|g(z)| < |f(z)|$ on C then show that $f(z)$ and $f(z) + g(z)$ both have same number of zeros inside C.

6) Explain Milne-Thomson Method for construction of an analytic function by considering both cases.

7) If $f(z) = \frac{x^3(1+i) - y^3\,(1-i)}{x^2 + y^2}$, $z \neq 0$ and $f(z) = 0,\ z = 0$ then show that $f(z)$ is continuous and satisfy Cauchy Riemann equations at origin and $f'(0)$ does not exist.

8) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles $z_1, z_2, \dots, z_n$ within the closed contour and continuous on boundary which is a rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^{n} Res(z = z_k)$ hence evaluate $\int_C \frac{e^{az}}{z+1}\, dz$ over the circle $C: |z| = 2$.

9) State and prove Cauchy theorem for simply connected domain.

10) State and prove sufficient conditions for $f(z)$ to be analytic.

11) If a domain D is bounded by system of closed rectifiable curves $C_1, C_2, \dots\dots, C_k$ and $f(z)$ is analytic in domain D and continuous on $C_1, C_2, \dots\dots, C_k$ then show that $\int_{C_1} f(z)dz + \int_{C_2} f(z)dz + \dots\dots\dots\dots\dots + \int_{C_k} f(z)dz = 0$

12) If $f(z)$ is analytic in domain D then show that $f(z)$ has derivatives of all orders at any point $z = a$ and all of which are analytic in domain D, there values is given by $f^n(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}}\, dz$ where C is any closed curve surrounding the point $z = a$.

13) State and prove Cauchy integral formula for simply connected domain.

14) Explain the method to evaluate the integral of the type $\int_{-\infty}^{\infty} f(z)dz$ hence show that $\int_0^{\infty} \frac{dz}{1+z^2} = \frac{\pi}{2}$.

15) State and prove Cauchy Residue theorem.

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