B. Sc. Part - II

1. **Title:** Mathematics
2. **Year of Implementation:** The syllabus will be implemented from June, 2019 onwards.
3. **Duration:** The course shall be a full time.
4 .**Pattern:** Semester examination.
5. **Medium of Instruction:** English.
6. **Structure of Course:**

B.Sc.-II Semester - III

Sr. No.	Paper Title	Theory			Practical		
		Paper Code	Lectures per week	Credits	Paper Title	Lectures per week	Credits
1	Real Analysis-I	BMT301	6	2	Practical Paper-III : BMP303	8	4
2	Algebra I	BMT302		2			

B.Sc.-II Semester - IV

Sr. No.	Paper Title	Theory			Practical		
		Paper Code	Lectures Per week	Credits	Paper Title	Lectures Per week	Credits
1	Real Analysis-II	BMT401	6	2	Practical Paper – IV : BMP403	8	4
2	Algebra-II	BMT402		2			

B: B.Sc. M: Mathematics T: Theory, P: Practical

3. Titles of papers of B.Sc. course: B.Sc.-II Semester-III

Theory: 45 lectures,36 hours(for each paper)

BMT301: Real Analysis-I

BMT302: Algebra-I

Practical: 80 lectures, 64hours

BMP303: Practical III

B.Sc. – II Semester – IV

Theory: 45 lectures,36 hours(for each paper)

BMT401: Real Analysis-II

BMT402: Algebra-II

Practical: 80 lectures, 64 hours Practical: BMP403: Practical IV

B.Sc. Part II Sem-III
BMT-301: Real Analysis-I (Credit:02)

Course Objective : Student Should :

1. understand basic statements and able to write basic proofs according to principles of quantificational logic.
2. understand thoroughly and precisely the concept of "limit" in its various forms.
3. define functions between sets equivalent sets, finite, countable and uncountable sets.
4. show whether sequence is converges or diverges.

Unit - 1 : Sets and Functions **[14]**

1.1 **Sets**

1.1.1 Operations on sets ,Cartesian product of sets ,Relation

1.2 **Functions**

1.2.1 **Definitions :** Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function

1.2.2 Theorem : If f : A $\rightarrow$ B and if X $\subseteq$ B, Y $\subseteq$ B then

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$$

1.2.3 Theorem : If : A $\rightarrow$ B and if X $\subseteq$ B , Y $\subseteq$ B then

$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$$

1.2.4 Theorem : If f : $\rightarrow$ B and X $\subseteq$A, Y $\subseteq$ A then $f(X \cup Y) = f(X) \cup f(Y)$

1.2.5 Theorem : If f : A $\rightarrow$B and X $\subseteq$ A, Y $\subseteq$ A then $f(X \cap Y) \subseteq f(X) \cap f(Y)$

1.2.6 **Definitions :** Injective, Surjective and Bijective function (1-1 correspondence), Inverse function

1.2.7 **Theorem :** Composition of two bijective functions is a bijective function

1.3 Countable Sets

1.3.1 **Definitions:** Finite sets, Infinite sets, Countable Sets, Uncountable Sets.

1.3.2 **Examples of Countable sets :** Set of Natural numbers, Set of Integers, Cartesian product of Countable sets

1.3.3 **Theorem :** Countable union of countable set is countable

1.3.4 **Theorem :** Set of Rational numbers is countable

1.3.5 **Theorem :** Any subset of countable set is countable

1.3.6 **Theorem :** The closed interval $[0,1]$ is uncountable

1.3.7 **Theorem :** The set of all real numbers is uncountable
1.3.8 **Theorem :** Every infinite set has a countably infinite subsets
1.3.9 **Examples**

Unit - 2 Completeness Property of R [08]

2.1 Definitions : Lower bound ,Upper bound of a subset of R , Bounded set, Supremum (*l*.u.b) , Infimum

2.2 Least Upper Bound Axiom [Completeness Property of]

2.3 Theorem (Archimedean Property) : If $x \in R$ then there exists $n_x \in N$ such that $x \le n_x$

2.3.1 Corollary : If $S = \left\{ \frac{1}{n} : n \in N \right\}$ then inf S = 0

2.3.2 Corollary : If t > 0 then there exist $n_t \in N$ such that $0 < \frac{1}{n_t} < t$

2.3.3 Corallary : If y > 0 then there exist $n_y - 1 < y < n_y$

2.4 Theorem : There exists a positive real number x such that $x^2 = 2$

2.4.1 Corollary : If x and y are real number x such that x < y then there exist an irrational number z such that x < z < y

2.5 Intervals

2.5.1 Characterization Theorem : If S is a subset of R that contains at least two points and has the property

If x, y $\in S$ and x < y then the closed interval [x, y] $\subseteq$ S then S is an interval

Unit -3: Sequence of Real Numbers

3.1 Sequence and Subsequence [14]

3.1.1 Definition of Swquence and Subsequence

3.2 Limit of a Sequence

3.2.1 Definition

3.2.2 Theorem : If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non - negative numbers and if $\lim_{n\to\infty} S_n = L$ *then* $L \ge 0$

3.3 Convergent Sequence

3.3.1 Theorem : Convergent sequence cannot converge to two distinct points

3.3.2 Theorem (Without Proof) : If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then any subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L

3.4 Operations on Convergent sequences.

3.4.1 Theorem : If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers.

If $\lim_{n\to\infty} S_n = L$ *and* $\lim_{n\to\infty} t_n = M$ *then* $\lim_{n\to\infty} (S_n + t_n) = L + M$

3.4.2 Theorem : If $\{S_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,

If $\lim_{n\to\infty} S_n = L$ *and* $\lim_{n\to\infty} t_n = M$ *then* $\lim_{n\to\infty} (S_n - t_n) = L - M$

3.4.3 Theorem : If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers,

If $c \in R$ and if $\lim_{n\to\infty} S_n = L$ *then* $\lim_{n\to\infty} CS_n = CL$

3.4.4 Theorem : If 0 < x < 1 then the sequence $\{x^n\}$ converges to 0

3.4.5 Lemma : If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then $\{S_n{}^2\}_{n=1}^{\infty}$ converges to L^2

3.4.6 Theorem : If $\{S_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,

If $\lim_{n\to\infty} S_n = L$ *and* $\lim_{n\to\infty} t_n = M$ *then* $\lim_{n\to\infty} (S_n .\ t_n) = LM$

3.4.7 Theorem : If $\{S_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,

If $\lim_{n\to\infty} S_n = L$ *and* $\lim_{n\to\infty} t_n = M$ *then* $\lim_{n\to\infty} (s_n / t_n) = L / M$

Unit - 4 : Monotone Sequences and Cauchy Sequences [09]

4.1 **Monotone Sequence**

4.1.1 **Definition and Examples**

4.1.2 **Theorem :** A non-decreasing sequence which is bounded above is convergent

4.1.3 **Theorem :** A non-increasing sequence which is bounded below is convergent

4.1.4 **Corollary :** The sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is convergent

4.1.5 **Theorem (Without Proof) :** A non-decreasing sequence which is not bounded above diverges to infinity

4.1.6 **Theorem (Without Proof) :** A non-increasing sequence which is not bounded below diverges to infinity

4.1.7 **Theorem :** A bounded sequence of real numbers has convergent subsequence

4.2 Cauchy Sequence

4.2.1 **Definition and Examples**

4.2.2 **Theorem :** If sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence

4.2.3 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$ is bounded

4.2.4 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$ is Convergent.

Course Outcomes :

Unit – I Sets and Functions.

After completion of the unit, **Students are able to**

1. define relation.
2. find domain, co-domain and range of given function.

Unit – II: **Completeness Property of R.**

After completion of the unit, **Students are able to**

1. find upper and lower bounds of subset of R.
2. identify any type of interval.

Unit – III: **Sequence of Real Numbers.**

After completion of the unit, **Students are able to**

1. define sequence and subsequence.
2. check given sequence is convergent or divergent.

Unit – IV: **Monotone Sequences and Cauchy Sequences.**

After completion of the unit, **Students are able to**

1. understand monotonic sequence.
2. define Cauchy sequence.

Reference Books :

1. **Methods of real Analysis ,**
   R.R.Goldberg, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi.

2. **Mathematical Analysis**,
   S.C.Malik and SavitaArora, New Age International (P) Limited, (Fifth Edition), 2017

3. **Calculus ,**
   T. M. Apostol, John Wiley and sons (Asia) P.Ltd, (Vol.I),2002

4. **Introduction to Real Analysis,**
   R.G.Bartle and D.R.Sherbert, Wiley India Pvt. Ltd, Fourth Edition, 2016

5. **First Course in Mathematical Analysis**,
   D.Somasundaram and B Choudhary, Narosa publishing house New, Delhi, Eighth Reprint, 2013.

6. **An Introduction to Real Analysis,**
   P.K.Jain and S.K.Kaushik, S.Chand&Company Ltd. New Delhi, First Edition, 2000

7. **Elements of Real Analysis,**
   Shanti Narayan and M.D.Raisinghania, S.Chand & Company Ltd. New Delhi, Fifteenth Revised Edition, 2014

******

B.Sc. Part II Sem-III

BMT - 302: Algebra - I (Credit : 02)

Course Objective : Student Should:

1. understand types of Matrices and their applications
2. develop the skills find the Eigen values and Eigen vectors
3. present the divisibility and relationship between the Greatest common divisor and least common multiple
4. define Types of Matrices, Divisibility in Integers, Equivalence relation and partitions and Congruence relation
5. present the concept Group and its basic properties

Unit - 1: Matrices **[12]**

1.1 **Introduction**

1.1.1 Definition with Illustration.

1.1.2 Types of Matrices

1.1.3 Definitions: Transpose of Matrix, conjugate of Matrix, Symmetric Matrix , Asymmetric Matrix

1.2 Hermitian and Skew hermitian

1.2.1 **Definitions :**Hermitian and Skew Hermitian

1.2.3 **Theorem:** The necessary and sufficient condition for a matrix A to be Hermitian is that $A = A^{\theta}$.

1.2.4 **Theorem:** The necessary and sufficient condition for a matrix A to be Skew Hermitian is that $A^{\theta} = -A$

1.2.5 **Theorem :** If A and B are Hermitian (Skew Hermitian) then A + B is also Hermitian (SkewHermitian).

1.2.6 **Theorem :** If A is Hermitian then iA is Skew Hermitian.

1.2.7 **Theorem :** If A is Skew Hermitian then iA is Hermition.

1.2.8 **Theorem :** Every square Matrix is uniquely expressed as the sum of Hermitian Skew hermitian matrix.

1.3 Eigenvalues and Eigenvectors

1.3.1 **Definitions :** Minor of matrix, Rank of a matrix, Inverse of a matrix, Characteristics Polynomial of matrix

1.3.2 **Eigenvalues and Eigenvectors**

1.3.3 Examples on 1.3.2.

1.4 System of a linear Equations

1.4.1 System of Homogeneous linear Equations.

1.4.2 Nature of solutions of AX = 0.

1.4.3 Examples on 1.4.1.

1.4.4 System of Non Homogeneous linear Equations.

1.4.5 Nature of Solution.

1.4.6 Examples on 1.4.4.

1.5 Cayley-Hamilton Theorem (Statement only)

1.5.1 **Applications of Cayley-Hamilton Theorem**.

Unit - 2 : Divisibility in Integers **[10]**

2.1 Definition :Divisibility in integers

2.2 The well ordering principle(Statement Only)

2.3 Properties of Divisibility

2.3.1 Definition of divisor and Multiple

2.3.2 **Theorem:**Let a,b,c,d be integers. Then

i] If a | b then a | bx

ii] If a | b and a | c then a| bx+cy $\forall$ x, y $\in I$

iii] If a | b and b | c then a | c

iv] If $m \neq 0$ is in Z and a | b $\Rightarrow$ am | bm

v] If a | b and c | d then ac | bd

vi] If ab | bc then a | c ; (b $\neq 0$)

2.4 Theorem: Division Algorithm (Without Proof)

2.5 Greatest common divisor and least common multiple

2.5.1 **Definitions:**Greatest common divisor and least common multiple

2.5.2 **Theorem :** Let a and b be two integers at least one of them not 0. Then there exist aunique greatest common divisor d of a and b. Moreover, d can be written as d = am + bn for integers m and n.

2.5.3 **The Euclidean Algorithm and Examples.**

2.5.4 Definition : Relatively Prime

2.5.5 **Euclid's lemma :** For a prime number p , if p | ab then either p | a or p | b.

2.6 Theorem : (Unique Factorization Theorem or Fundamentals Theorem of Arithmetic)

Unit - 3 : Relation [12]

3.1 Relation

3.1.1 **Definitions:** Cartesian Product, Relation, Binary Relation, Inverse Relation.

3.1.2 Examples on 3.1.1.

3.2 Pictorial Representation of Relation

3.2.1 Co-ordinate Diagram

3.2.2 Arrow Diagram

3.2.3 Matrix Representation

3.2.4 Directed Graph

3.2.5 Examples on 3.2.1 to 3.2.4

3.3 Composition of Relations

3.3.1 **Definition:** Composition of Relations

3.3.2 **Theorem:** Let A, B, C and D be sets. Let R: A $\rightarrow$B , S: B$\rightarrow$C , T: C $\rightarrow$D be Relation then $R \circ (S \circ T) = (R \circ S) \circ T$

3.4 Types Of Relations

3.4.1 **Definitions :** Reflexive, Symmetric, Antisymmetric and transitive.

3.4.2 Examples on 3.4.1

3.4.3 **Theorem:** Let R be relation on set A. Then R^{∞} is the smallest transitive relation on **A** that contain R.

3.5 Equivalence relation and partitions

3.5.1 **Theorem:** Let R be an equivalence relation on set A. Then quotient set A/R forms a partition of A.

3.5.2 **Theorem:** Let $\{A_i\}, i \in I$ be partition of a set A. Then there exists an equivalence Relation on the set A such that quotient set A / R is the given partition $\{A_i\}, i \in I$ *on A*.

3.6 Partial order relation.

3.6.1 **Definition**: Partial order relation.

3.6.2 Examples on 3.6.1

3.7 Congruence relation on Integers

3.7.1 **Definition** : Congruence relation

3.7.2 **Congruence arithmetic**

3.7.3 **Theorem:** Let n > 1 be a fixed positive integer and a, b, c, d be arbitrary integers then the following conditions holds

i] If $a \equiv b \ (\text{modn})\, and\, c \equiv d \,(\text{modn})\, then a + c \equiv b + d \,(\text{modn})$.

ii] If $a \equiv b \ (\text{modn})\, and\, c \equiv d \,(\text{modn})\, then \ \ ac \equiv bd \,(\text{modn})$.

3.7.4 **Examples**

Unit - 4 : Groups [11]

4.1 Binary operation on a set

4.1.1 **Definition** :Binary operation on a set with illustration

4.2 Semigroup

4.2.1 **Definition :** Semigroup with illustration

4.3 Monoid

4.3.1 **Definition :**Monoid with illustration

4.4 Group

4.4.1 **Definition:** Group, Abelian Group, Finite Group, Infinite Group ,Order of a group

4.4.2 Examples on 4.4.1

4.5 Properties of Groups

4.5.1 **Theorem :** If < G, * > is a group, then

a] Identity element in G is unique

b] Every α in G has unique inverse in G.

c] For every a in G, $\left(a^{-1}\right)^{-1} = a$

d] For all a, b $\in$ G , $(a*b)^{-1} = b^{-1} * a^{-1}$

4.5.2 **Theorem :** If a, b, c are elements in a group G, then

i] a* b = a * c implies b = c (Left Cancellation Law)

ii] b * a = c * a implies b = c (Right Cancellation Law)

4.5.3 **Theorem :** If G is a group and a,b $\in$ G, then the equations a * x = b and y * a = b have unique solutionsn $x = a^{-1} * b$ *and* $y = b^{-1} * a$ respectively.

4.5.4 **Definition :** Order of element with illustration, Properties(Without Proof)

4.6 Permutations

4.6.1 **Definition with Illustration**

4.6.2 **Cyclic Permutation**

4.6.3 **Transposition , Disjoint Permutations , Even and Odd Permutations**

Course Outcomes:

After completion of the unit, students are able to:

Unit – I Matrices.

1. define Transpose of Matrix, conjugate of Matrix, Symmetric Matrix , Asymmetric Matrix
2. define Hermitian matrix, Skew Hermitian matrix, Minor of matrix, Rank of a matrix, Inverse of a matrix,Characteristics Polynomial of matrix, Eigen values and Eigenvectors.

Unit – II. Divisibility in Integers.

1. define divisibility in integers, divisor and Multiple, Greatest common divisor and least common multiple.
2. understand Well Ordering Principle and Euclid's Lemma.

Unit – III. Relation.

1. define Relation, Equivalence Relation, Partial Order Relation and Congruence Relation.
2. understand pictorial representation and types of Relations.

Unit – IV. Group.

1. define binary operations, Semigroup, Monoid and Group.
2. understand properties of Groups and concept of Permutation Groups .

Reference Books :

1. **A first couse in abstract Algebra**, J.B. Fraleigh, Narosa Publishing House New Delhi, Tenth Reprint, 2003
2. **Modern Algebra**, A. R.Vasishtha, Krishna Prakashan, Meerut 1994
3. **Algebra,** M. Artin, Prentice hall of India, New delhi, 1994
4. **Topics in Algebra**, I.N. Herstein , Wiley India Pvt. Ltd.
5. **A Text Book of Matrices**, Shantinarayan, S. chand Co., Pvt. Ltd. Raminagar, New Delhi.
6. **Elementarary Number Theory,** David. M. Burton**,** McGraw Hill Education, 7^{th} Edition, 2017
7. **Discrete Mathematics**, Schaum's Outline, Tata MaGraw - Hill Publishing Company Ltd., New Delhi. (Unit), 3^{rd} Edition.
8. **A course in abstract Algebra** , V.K. Khanna and S. K. Bhambri, Vikas Publishing house Private Limited, New Delhi, Fifth Edition, 2016.

******

BMP 303: Practical Paper III
(Credits: 04)

Course Objectives : Student should

1. Understand how matrices and determinants are used as mathematical tools in QA.
2. Find power of Matrix and determining the inverse of matrix.
3. Use Matrices to represent a system of equations.
4. Provide exposure to problem solving through programming.
5. Apply basic concepts of the c-programming language.

Group - A

Sr. No.	Name of the practical	No. of Practical's
1.	Solution of System of m linear homogeneous equations in n unknowns	1
2.	Solution of System of m linear non homogeneous equations in n unknown	1
3.	Inverse of Matrix by Cayley Hamilton Method	1
4.	Euclidean Algorithm	1
5.	Pictorial Representation of Relation	1
6.	Examples on equivalence relation	1
7.	Examples on Fermat's theorem	1
8.	Examples on Group & Order of an element	1
9.	Beta function	1
10.	Gamma function	1

Group - B

11.	C-Introduction - I	1
12.	C-Introduction - II	1
13.	Complete Structure of C - programe	1
14.	Simple C- programe	1
15.	If Statement, If else Statement & Switch Statement	1
16.	While loop & do while loop	1
17.	For loop	1
18.	Go to, break continue statement	1
19.	One Dimensional Array	1
20.	Two Dimensional Array	1

Course Outcomes:

Group A- after completion, students are able to:

1. critically analyze and construct Mathematical arguments that relate to the study of introductory Matrix theory.
2. develop the applications of Matrix theory.
3. recognize the types of group when described using a standard forms.

Group - B

After completion, students are able to:

1. develop logics which will help them to create programs.
2. use if statement, if else statement ,for loop, do while loop in coding.
3. write a code using one dimensional and two dimensional array.

Reference Books:

1. **A Text Book of Matrices**
   Shantinarayan, S. chand Co., Pvt. Ltd. Raminagar, New Delhi.
2. **A course in abstract Algebra**
   V.K. Khanna and S. K. Bhambri, Vikas Publishing house Private Limited, New Delhi, Fifth Edition, 2016.
3. **A Hand Book of Computational Mathematics Laboratory**
   R.B. Kulkarni, U.H. Naik, J.D. Yadhav, S.P. Thorat, A. A. Basade, H.V. Patil, H.T. Dinde, Shivaji University Mathematics Society, 2005.
4. **Integral Calculus**
   Shanti Narayan, P.K. Mittal, S. Chand and comp, New Delhi.
5. **Computational Mathematics**
   B.P. Demidovich & I. A. Maron, translated by George Yankosky, Mir Publishers, Moscow.
6. **A first couse in abstract Algebra**
   J.B.Fraleigh, Narosa Publishing House New Delhi, Tenth Reprint, 2003.
7. **Modern Algebra**
   A. R. Vasishtha , Krishna Prakashan , Meerut 1994.
8. **Schaum's Outline**
   Seymour hipschutz, Marehipson, Discrete Mathematics, Tata MaGraw-Hill Publishing Company Ltd., New Delhi, (3rd Edition).

******

B. Sc. –II: Mathematics Semester – IV
BMT-401: Real Analysis-II (Credits: 2)

Course Objectives : Students should

1] work within an axiomatic framework.

2] use basic ideas of Real Analysis and it is required for their subsequent course work.

3] define Limit Superior and Inferior of Sequences and tests for convergence of series.

4] apply sequence and series of functions especially useful in obtaining approximations to a given function and defining new functions from known ones.

5) understand sequences whose terms are functions rather than real numbers and pay attention to the general properties that are associated with the uniform convergence of sequence and series of functions.

Unit - 1: Limit Superior and Inferior of Sequences [14]

1.1 **Definitions and Examples**

1.1.1 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then

$$\lim_{n\to\infty} \sup\ s_n = \lim_{n\to\infty} s_n$$

1.1.2 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then

$$\lim_{n\to\infty} \inf\ s_n = \lim_{n\to\infty} s_n$$

1.1.3 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers and if

$$\lim_{n\to\infty} \sup\ s_n = \lim_{n\to\infty} s_n \inf\ s_n = L \textit{ where } L \in R \textit{ then } \{s_n\}_{n=1}^{\infty}$$

is convergent and $\lim_{n\to\infty} s_n = L$

1.1.4 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers and if $s_n \le t_n$ then

i] $\lim_{n\to\infty} \sup\ s_n \le \lim_{n\to\infty} \sup\ t_n$ ii] $\lim_{n\to\infty} \inf\ s_n \le \lim_{n\to\infty} \inf\ t_n$

1.1.5 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then

i] $\lim_{n\to\infty} \sup(s_n + t_n) \le \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$

ii] $\lim_{n\to\infty} \inf(s_n + t_n) \ge \lim_{n\to\infty} \inf s_n + \lim_{n\to\infty} \inf t_n$

Unit - 2 : Series of Real Numbers [09]

2.1 Convergent and Divergent Series

2.1.1 **Definitions :** Convergent Series, Divergent Series and Examples

2.1.2 If $\sum_{n=1}^{\infty} a_n$ is convergent series then $\lim_{n\to\infty} a_n = 0$

2.2 Cauchy's General Principal for convergence(Statement only)

A necessary and sufficient condition for the convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that the sequence of its partial sum $\{s_n\}$ is convergent

2.3 Series of Nonnegative real numbers

2.3.1 **Definition and Examples**

2.3.2 **Theorem :** A positive term series converges if and only if its sequence of partial sum is bounded above

Theorem : A positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if p > 1

2.4 Tests for convergence

2.4.1 Comparison Test (First Type)

If $\sum u_n$ *and* $\sum v_n$ are two positive term series and $k \neq 0$, a fixed positive real number (independent of n) and there exists a positive integer m such that $u_n \le kv_n$ for every $n \ge m$ then

a] $\sum u_n$ is convergent if $\sum v_n$ is convergent and

b] $\sum v_n$ is divegent if $\sum u_n$ is divergent

2.4.2 Comparison Test (Second Type)

If $\sum u_n$ and $\sum v_n$ are two positive term series and there exist positive number m, such that

$(u_n / u_{n+1}) \geq (v_n / v_{n+1})$ *for every* $n \geq m$ *then*

a] $\sum u_n$ is convergent if $\sum v_n$ is convergent and

b] $\sum v_n$ is divegent if $\sum u_n$ is divergent

2.4.3 p - Series Test : The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$

2.4.4 Root Test : Consider the series $\sum_{n=1}^{\infty} a_n$. Then

a] If lim sup $|a_n|^{1/n} < 1$ then the series convergent absolutely.

b] If lim sup $|a_n|^{1/n} > 1$ then the series diverges

c] If lim sup $|a_n|^{1/n} = 1$, this test gives no information

2.5 Alternating Series

2.5.1 Leibnitz Test : If the alternating series

$u_1 - u_2 + u_3 - u_4 + ...,(u_n > 0 \text{ for every } n)$ is such that

i] $u_{n+1} \leq u_n$, for every n and

ii] lim $u_n = 0$ then the series converges

2.6 Examples

2.7 Absolute and Conditional Convergence

2.7.1 Definition and Examples

2.7.2 Theorem : Every Absolutely convergent series is convergent

2.8 Examples

Unit - 3 : Sequence and Series of Functions [14]

3.1 Pointwise convergence of sequence of functions

3.1.1 Definition and Examples

3.2 Uniform convergence of sequence of functions

3.2.1 Definition and Examples

3.3 Uniform Convergence and Continuity

3.3.1 Theorem : Assume $f_n \to f$ uniformly on an interval S. If each function f_n is continuous at a point p in s then the limit function f is also continuous at p

3.3.2 Theorem : If series of functions $\sum u_k$ converges uniformly to a function f on a set S and if each term u_k is continuous at a point p in s then f is also continuous at p

Unit - 4 : Differentiability and Integrability of Series of functions [08]

4.1 Theorem : Assume that $f_n \to f$ uniformly on an interval [a, b] and assume that each function f_n is continuous on [a, b]. Define a new sequence {g_i} by the equation $g_n(x) = \int_a^x fn(t)\, dt\, if\;\; x \in [a,b]\, and\;\; g(x) = \int_a^x f(t)\, dt\;\; Then\, g_n \to g$ uniformly on [a, b]. In symbols, we have

$$\lim_{n\to\infty} \int_a^x f_n(t)\,dt = \int_a^x \lim_{n\to\infty} f_n(t)\,dt$$

4.2 Theorem : Assume that series of fucntions $\sum u_k$ converges uniformly to a function f on an interval [a, b] where each u_k is continuous on [a, b]

for $x \in [a,b]$

define $g_n(x) = \sum_{k=1}^{n} \int_a^x u_k(t)\, dt\;\; and\;\; g(x) = \int_a^x f(t)\,dt$

Then $g_n \to g$ uniformly on [a, b]

4.3 Sufficient Condition of Uniform Convergence

4.3.1 Theorem (Weierstrass M - Test) : Given series of functions $\sum u_k$ which converges pointwise to a function f on a set s. If there is a convegent series of positive constants $\sum M_n$ such that $0 \le |u_n(x)| \le M_n\, for\;\, every\;\, n \ge 1$ and every x in S. Then $\sum u_k$ converges uniformly on S.

4.4 Power Series

4.4.1 Definition

4.4.2 Interval of Convergence and its examples.

Course Outcomes:

Unit - I : After completion of the unit, Students are able to

1. define limit superior and limit inferior .
2. understand difference between superior and inferior and solve example.

Unit - II : After completion of the unit, Students are able to

1. state various test for checking the convergence.
2. apply various tests to convergence of series.

Unit - III : After completion of the unit, Students are able to

1. define uniform and point wise convergence.
2. understand test of uniform convergence

Unit - IV: After completion of the unit, Students are able to

1. define power series and interval of convergence.
2. understand properties of uniform convergence.

Reference Books :

1. **Methods of real Analysis**
   R.R.Goldberg , Oxford & IBH Publishing co. Pvt. Ltd, New Delhi
2. **Mathematical Analysis**
   S.C.Malik and SavitaArora, (Fifth Edition), New Age International (P) Limited, 2017.
3. **Calculus**
   Tom M Apostol, John Wiley and sons (Asia) P.Ltd.2002
4. **Introduction to Real Analysis**
   R.G. Bartle and D.R.Sherbert, Wiley India Pvt. Ltd, 4^{th} Edition, 2016
5. **First Course in Mathematical Analysis**
   D.Somasundaram and B Choudhary, Narosa publishing house New, Delhi, and Eighth Reprint 2013.
6. **An Introduction to Real Analysis**,
   P.K.Jain and S.K.Kaushik, S.Chand & Company Ltd. New Delhi, 1^{st} Edition, 2000
7. **Elements of Real Analysis**,
   Shanti Narayan and M.D.Raisinghania, S.Chand & Company Ltd.New Delhi, Fifteenth Revised Edition, 2014
8. **A course of Mathematical Analysis**,
   Shanti Narayan andP.K.Mittal, S.Chand & Company Ltd. New Delhi, Reprint 2016

******

(BMT - 402) Algebra - II
(Credits 02)

Course Objective : Student Should

1] understand types of subgroups and how to identify them.
2] develop the skills to use various groups and to prove various results.
3] present the relationship between abstract algebraic structures with familiar group theory
4] define Subgroups, Normal subgroup, Cyclic Subgroups, Homomorphism and Permutation Group.
5] present the concept of Kernel of Homomorphism and Permutation Group structure.

Unit - 1 : Subgroups [12]

1.1 **Subgroups**

1.1.1 **Definition :** Subgroups with illustrations

1.2

1.2.1 **Theorem :** A non empty subset H of a group G is a subgroup of G If and only if

i] $a, b \in H \Rightarrow ab \in H$

ii] $a \in H \Rightarrow a^{-1} \in H$

1.2.2 **Theorem :** A non empty subset of a group G is a subgroup of G iff

$$a, b \in H \Rightarrow ab^{-1} \in H$$

1.2.3 **Theorem :** A non empty finite subset H of a group G is a subgroup of G iff H is Closed under multiplication.

1.3 **Centre of a Group**

1.3.1 **Definition:** Centre of a Group ,Normalizer of a element with illustration.

1.3.2 **Theorem :** Centre of group G is subgroup of group G.

1.3.3 **Theorem :** Normalizer of an element group G is subgroup of group G.

1.4 Cosets

1.4.1 **Definition:** Coset and examples

1.4.2 Let H be a subgroup of G then

i] $Ha = H \Leftrightarrow a \in H \text{ and } aH = H \Leftrightarrow a \in H$

ii] $Ha = Hb \Leftrightarrow ab^{-1} \in H \text{ and } aH = bH \Leftrightarrow a^{-1}b \in H$

iii] Ha (aH) is a subgroup of G iff $a \in H$

1.3.3 **Theorem :** $Ha = \{x \in G \mid x \equiv a \bmod H\} = cl(a)$ for any a in G

1.4 Lagrange's Theorem

1.4.1 Theorem: If G is a finite group and H is a subgroup of G then o(H) divides o(G)

1.5 Index of a subgroup

1.5.1 **Definition :** Index of subgroup H in G with illustration

1.6 **Theorem :** For subgroups H and K of G ,HK is a subgroup of G iff HK = KH

Unit - 2 : Cyclic groups [11]

2.1 **Cyclic groups**

2.1.1 **Definition :** Cyclic group , generator of a cyclic group

2.1.2 Examples on 2.1.1

2.2

2.2.1 **Theorem:** Order of a cyclic group is equal to the order of its generator.

2.2.2 **Theorem:** A subgroup of cyclic group is cyclic.

2.2.3 **Theorem :** Every cyclic group is abelian.

2.2.4 **Theorem :** If G is finite group then order of any element of G divides order of G.

2.2.5 **Theorem :** An infinite cyclic group has precisely two generators.

2.3 Euler ϕ function

2.3.1 **Definition :** Euler's ϕ function

2.3.2 **Theorem :** Number of generators of a finite cyclic group of order n is ϕ(n).

2.4 Euler and Fermat's Theorem

2.4.1 **Euler's Theorem :** Let $a,\ n(n \geq 1)$ be any integers such that gcd (a, n) = 1 then

$$a^{\phi(n)} \equiv 1 \text{ (modn)}$$

2.4.2 **Fermat's Theorem :** For any integer a and prime p $a^p \equiv a(\text{modp})$

2.4.3 Examples on 2.4.1 and 2.4.3

Unit - 3 : Normal groups [11]

3.1 Normal groups

3.1.1 Definitions : Normal subgroups , Simple group

3.1.2 Examples

3.2 **Results on Normal Groups**

3.2.1 **Theorem :** A subgroup H of group G is normal in G $iff g^{-1}Hg = H,\ g \in G.$

3.2.2 **Theorem :** A subgroup H of group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H,\ g \in G$.

3.2.3 **Theorem :** A subgroup H of group G is normal in G iff the product of two right (left) cosets of H in G is again a right (left) coset of H in G.

3.3 Quotient groups

3.3.1 **Definition : Quotient groups with illustration.**

3.3.2 **Theorem :** If G is finite group and N is normal subgroup of G then $0\left(\frac{G}{N}\right) = \frac{0(G)}{0(N)}$.

3.3.2 **Theorem :** Every quotient group of cyclic group is cyclic.

Unit - 4 : Homomorphism , Permutation Group [11]

4.1 **Homomorphism**

4.1.1 Definitions: Homomorphism ,Epimorphism , Monomorphism , Endomorphism and Automorphism.

4.1.2 Examples on 4.4.1

4.1.3 **Theorem :** If $f : G \to G'$ is homomorphism then

i] f (e) = e

ii] $f\left(x^{-1}\right) = \left[f(x)\right]^{-1}$

iii] $f\left(x^{n}\right) = \left[f(x)\right]^{n}$, n is an integer.

4.2 **Kernel of Homomorphism**

4.2.1 **Definition:** Kernel of Homomorphism with illustration

4.2.2 **Theorem :** If $f : G \to G'$ is homomorphism then kerf is a normal subgroup of G.

4.2.3 **Theorem :** A homomorphism $f : G \to G'$ is one-one if and only if kerf= {e}.

4.3 **Isomorphism Theorems**

4.3.1 **Fundamental Theorem of group Homomorphism :** If $f : G \to G'$ is an onto homomorphism with K = kerf then $\frac{G}{K} \cong G'$.

4.3.2 **Second theorem of Isomorphism :(Statement only)** Let H and K be two subgroups of group G , where H is normal in G, then $\frac{HK}{H} \cong \frac{K}{H \cap K}$.

4.3.3 **Third theorem of Isomorphism (Statement only):** Let H and K be two normal subgroups of group G ,such that $H \subseteq K$ then $\frac{G}{K} \cong \frac{G/H}{K/H}$.

4.4 **Permutation Group**

4.4.1 **Cayley Theorem:** Every group G is isomorphic to a permutation group.

4.4.2 **Theorem:** Set of even permutations is a normal subgroup of S_n Alternating group.

Course Outcomes:

Unit - I : After completion of the unit, Students are able to

1. define subgroups ,centre of group.
2. understand Lagrange's theorem and apply it.

Unit - II : After completion of the unit, Students are able to

1. state Euler and Fermat's theorem.
2. understand cyclic group and give example.

Unit - III : After completion of the unit, Students are able to

1. define normal groups and quotient group.
2. understand index of group.

Unit - IV: After completion of the unit, Students are able to

1. define homomorphism of groups.
2. give examples of homomorphism and check the homomorphism

Reference Books :

1. **A course in abstract Algebra**
   V.K. Khanna and S. K. Bhambri, Vikas Publishing house Private Limited, New Delhi, 3rd Edition 2008.
   2. A first course in Abstract Algebra
   J.B.Fraleigh, Narosa Publishing House New Delhi, Tenth Reprint 2003.
3. **Modern Algebra**
   A.R. Vasishtha, Krishna Prakashan, Meerut 1994 .
4. **Algebra**
   M.Artin, Prentice hall of India, New delhi, 1994.
5. **Topics in Algebra**
   I.N. Herstein, Wiley India Pvt. Ltd.

****

BMP 403: Practical Paper IV
(Credits: 04)

Course Objective : Student Should:

1. develop the skills to use various groups and to prove various results.
2. fine double integration over rectangle.
3. use sequence and series of functions are especially useful in obtaining approximations to a given function and defining new functions from known ones.
4. use change variables in multiple integrals.
5. transform the system of equations to a new having upper triangular form which back substitution scheme.

Group - A

Sr. No.	Name of the practical	No. of Practical's
1	Examples on Cyclic Group	1
2	Examples on Normal Subgroup	1
3	Permutation Group	1
4	Homomorphism and Group	1
5	Comparison test and Cauchy's Root test	1
6	D'Almberts Ratio test and P-test	1
7	Double Integration over the given region	1
8	Double Integration : Change of order of integration	1
9	Double Integration : Change of co-ordinate axis	1
10	Double Integration by using Polar Co-ordinates	1

Group - B

11	Function	1
12	Trapazoidal Rule and its Program	1
13	Simpsons(1/3)rd rule and program	1
14	Simpsons(3/8)th rule and program	1
15	Gauss Elimination Method	1
16	Gauss Jordan Method	1
17	Gauss-Seidel Method	1
18	Euler's Method	1
19	Euler's Modified Method	1
20	Runge-Kutta second & fourth order Method	1

Learning outcomes:

Group A : After completion of the practical's are able to

1. develop the applications of Matrix theory.
2. explain Demonstrate accurate and efficient use of advanced algebraic techniques

Group B : After completion of the practical's are able to

1. solve the problems involving various methods of eliminations.
2. solve the system of linear equations by using numerical methods.

Reference Book:

1. **A Text Book of Matrices**
   Shantinarayan, ,S. chand Co.,Pvt. Ltd. Raminagar, New Delhi.
2. **A course in abstract Algebra**
   V.K. Khanna and S. K. Bhambri , Vikas Publishing house Private Limited ,New Delhi , Fifth Edition 2016.
3. **A Hand Book of Computational Mathematics Laboratory**
   R.B. Kulkarni, U.H. Naik, J.D. Yadhav, S.P. Thorat, A.A. Basade, H.V. Patil, H.T. Dinde, Shivaji University Mathematics Society, 2005.
4. **Integral Calculus**
   Shanti Narayan, P.K. Mittal :, S. Chand and comp. New Delhi.
5. **Computational Mathematics**
   B.P. Demidovich & I. A. Maron,translated by George Yankosky Mir Publishers, Moscow.
6. **A first couse in abstract Algebra**
   J.B.Fraleigh, Narosa Publishing House New Delhi, Tenth Reprint 2003.
7. **Modern Algebra**
   A.R. Vasishtha, Krishna Prakashan, Meerut 1994.
8. **Schaum's Outline , Discrete Mathematics**
   Seymour hipschutz, Marehipson, Tata MaGraw - Hill Publishing Company Ltd., New Delhi., (3rd Edition)

****